Neutrino Mixing And Cosmology

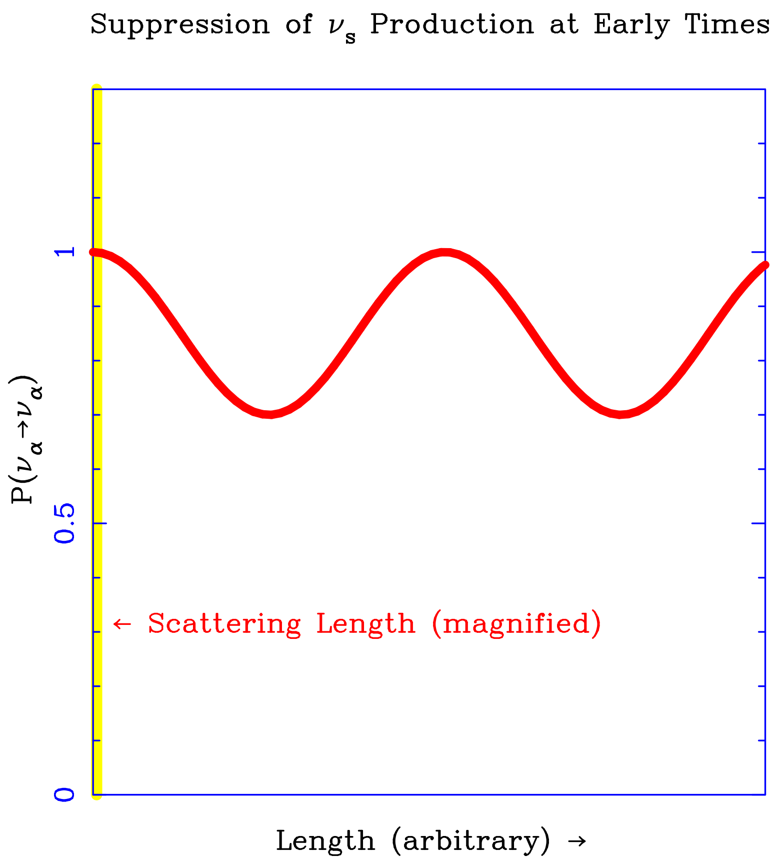
Kevork Abazajian
George M. Fuller

SLAC Summer Institute
August 21, 2000

Poster Presentation

The Quantum Zeno Effect

Each scattering of a neutrino acts as a “measurement” of the neutrino flavor state. The scattering length is extremely short compared to the oscillation length at high temperatures ($T \sim 150$ MeV). Therefore, an active neutrino will not have time to evolve a probability to become a sterile state.



As the universe expands, cools, and becomes less dense, the scattering rate goes down. Then, the probability for scattering “into” a sterile neutrino state goes up. Since the oscillation length is inversely proportional to δm^2, larger mass differences oscillate more quickly and transform to steriles more rapidly. Also, if $\sin^2 2\theta$ is large, the probability of scattering into a sterile state is greater. This is why oscillation-based constraints from the early universe exclude the upper right portion of a $\sin^2 2\theta$ vs. δm^2 plot, where these values are large.

MSW in the Early Universe:
Neutrino Resonance

The neutrino state evolution for an electron neutrino produced in the sun can be described by the pseudo-Schrödinger equation

$$i\hbar\frac{d}{dt}\begin{pmatrix}\nu_\alpha \\ \nu_s\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\delta m^2\cos 2\theta + A & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha \\ \nu_s\end{pmatrix},$$

if the solar solution is given by mixing with a sterile neutrino, and $A = 2\sqrt{2}G_F E(N_e - 0.5N_n)$.

The evolution of a neutrino state in the early universe is very similar (if one neglects collisions), but in the early universe (for $\nu_e \to \nu_s$ mixing)

$$A = 2\sqrt{2}G_F E N_\gamma (V^L + V^T).$$

Here,

$$V^L = 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} - \frac{1}{4}\eta$$

takes into account asymmetries in the matter/neutrinos and is positive (negative) for neutrinos (antineutrinos); $L_{\nu_\alpha} \equiv (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/n_\gamma$, and

$$V^T = -110\ G_F^2 E T^4 \frac{n_{\nu_e} + n_{\bar{\nu}_e}}{n_\gamma}$$

is unique to the hot & dense early universe, and results from considerations of the coherent cross-section in finite-temperature field theory.

Resonance occurs when the mass levels cross, or when $\delta m^2 \cos 2\theta = A$. Since A is momentum dependent, the resonance is as well. The resonance for neutrinos and antineutrinos is the same if there is no asymmetry, L_ν, but will be different for finite L_ν. The positions of the resonances can cause either a **stable evolution** or an **unstable evolution**.

Stability and Instability

As if there is a small matter asymmetry, the positions of the resonance (maximal mixing) in the distribution of the neutrinos will be different for neutrinos and antineutrinos due to the sign difference of V^L.

- **For Neutrinos:** the position (p/T) of resonance increases if L_ν increases.

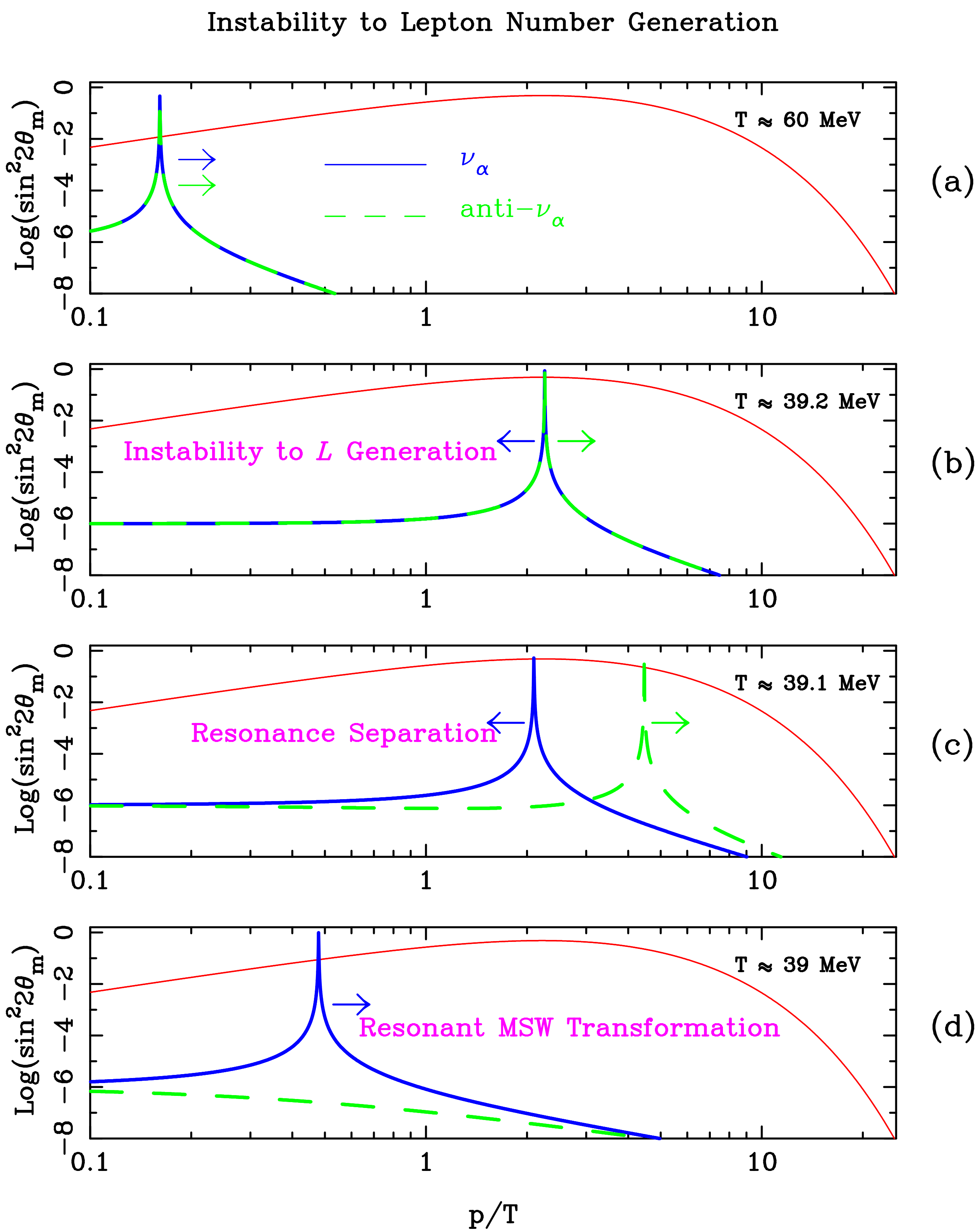
- **For Antineutrinos:** the position of the resonance decreases as L_ν increases.

This dependance causes a stability if the resonances are below the distribution peak ($p/t \sim 2.2$), and an instability if the resonances are above the distribution peak.

Below the distribution peak, the neutrino resonance samples a greater number of neutrinos than the antineutrino resonance (due to the greater abundance at higher p/T). This has the property of lowering L_ν, drives the resonances of $\nu/\bar{\nu}$ closer together, and stabilizes the evolution.

Above the distribution peak, the neutrino resonance samples a lower number of neutrinos than the antineutrino resonance (due to the lower abundance at higher p/T). This has the property of increasing L_ν, driving the resonances further apart, and amplifying the difference between the $\nu/\bar{\nu}$ resonances. This nonlinearly amplifies the magnitude of L_ν.

The same amplification occurs above the thermal peak for $L_\nu < 0$, as the positions of the $\nu/\bar{\nu}$ resonances are switched.



Instability to Lepton Number Generation
Log(sin²2θ_m)
T ≈ 60 MeV
ν_α
anti−ν_α
(a)
T ≈ 39.2 MeV
Instability to L Generation
(b)
T ≈ 39.1 MeV
Resonance Separation
(c)
T ≈ 39 MeV
Resonant MSW Transformation
(d)
0.1
1
10
p/T

Generation of Lepton Number

For a standard case where $\delta m^2_{se} < 0$, the evolution of the resonant conversion proceeds like the following. A schematic picture of the evolution is to the left, with steps (a)-(d) being:

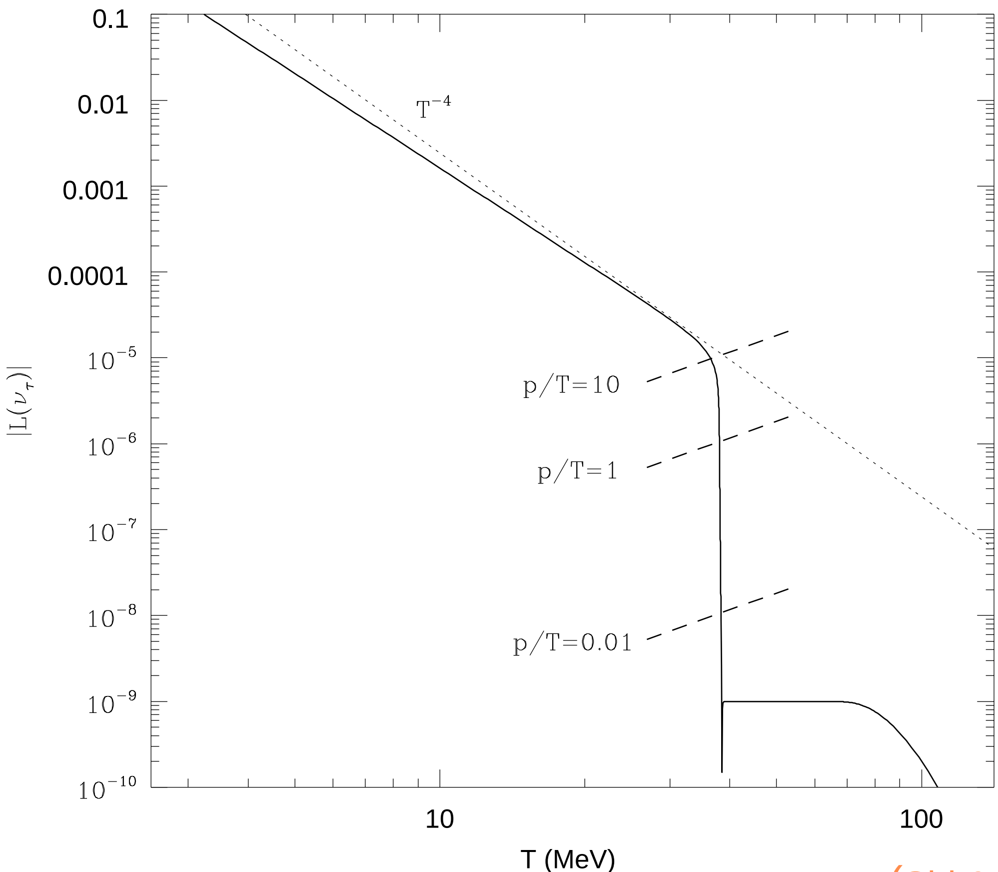
(a) Initially, one can assume that $L_{\nu_\alpha} = 0$, so the effective potential V is dominated by the thermal term V^T. Therefore, the ν_α and $\bar{\nu}_\alpha$ have the same resonance. The population of the sterile neutrinos proceeds symmetrically.

(b) At this point, just to the high energy side of the thermal distribution, the lower temperatures bring the V^L term closer to V^T. An instability sets in, and drives the production of lepton number since the resonances are sampling different portions of the distribution and have different efficiencies.

(c) The instability drives a separation of the neutrino and anti-neutrino resonances. The asymmetry L starts being produced nonlinearly.

(d) Only the neutrino resonance remains within the populated portion of the energy spectrum. A resonant MSW conversion of the active neutrinos to steriles proceeds. In this case, $L_\nu < 0$ is generated. It can be equally likely that the anti-neutrino resonance is left and $L > 0$ is generated.

Generation of Lepton Number II



(Shi & Fuller, 1999)

The magnitude of a lepton asymmetry will exponentially grow as the resonance sweeps through the energy spectrum of the active neutrino or antineutrino.

Adiabaticity

and the Efficiency of Resonant Conversion

In general, the resonant conversion of a neutrino from one flavor to another is only successful and complete when the resonance is adiabatically evolving. It is not generally the case that a resonance is adiabatic.

As with the solar solution, under certain conditions the conditions are non-adiabatic in the early universe, and resonant conversion can halt, or not happen at all. In a matter environment that is changing, resonant conversion travels through the energy distribution of the neutrino flavor. The distance over which the resonant conversion occurs is

$$\delta r = 2\left[-\frac{1}{\rho}\frac{d\rho}{dr}\right]^{-1}\tan 2\theta.$$

This can be parametrized as a length of time in the early universe. The oscillation length of the neutrino is

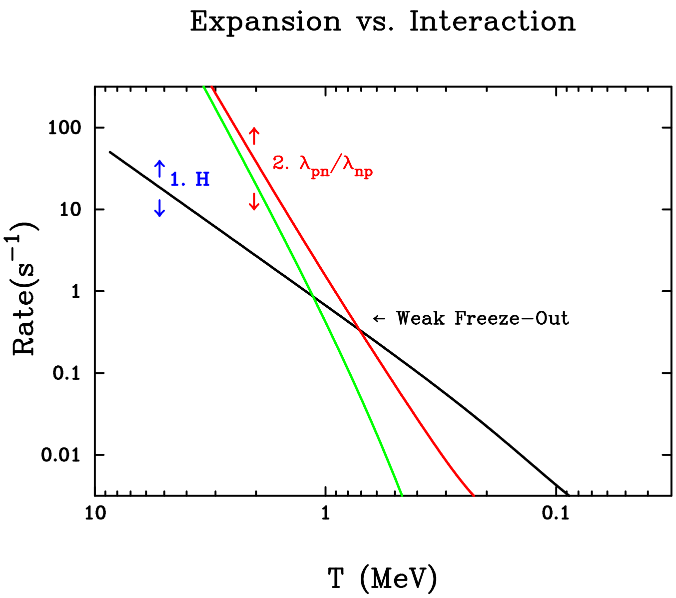
$$L_{\rm osc} = \frac{4\pi p}{\delta m_M^2}$$

where $\delta m_M^2 = \sqrt{(\delta m^2 \sin 2\theta)^2 + \left[\delta m^2 \cos 2\theta + 2pV_\alpha\right]^2}$ is the effective mass difference squared in matter.

The resonant conversion of one flavor to another is adiabatic and complete when the oscillation length is much smaller than the resonance width. (In the early universe, the oscillation period must be much smaller than the resonance time.) Therefore, the vacuum oscillation phase does not interfere with the matter effect. When the oscillation period is comparable to the resonance time, the “probability of hopping” (Cf. Dr. Murayama’s lectures) becomes likely.

A resonant conversion may become non-adiabatic and stop conversion of an active flavor to a sterile. It is just this effect that can produce non-thermal sterile neutrino dark matter (Shi & Fuller, 1999).

Cosmic Helium Production in Two Easy Steps:



1. **Hubble Expansion:**

$$
\begin{aligned}
H &\propto \rho^{1/2} \\
H &\propto (\rho_\gamma + \rho_{e^\pm} + \rho_{\nu_\alpha} + \rho_{\nu_s} + \ldots)^{1/2}
\end{aligned}
$$

2. **Weak Interaction Rates:**

$$
\lambda_{n\to p}\begin{cases} n + e^+ \to p + \bar{\nu}_e \\ n + \nu_e \to p + e^- \\ n \to p + e^- + \bar{\nu}_e \end{cases} \qquad \lambda_{p\to n}\begin{cases} p + \bar{\nu}_e \to n + e^+ \\ p + e^- \to n + \nu_e \\ p + e^- + \bar{\nu}_e \to n \end{cases}
$$

"Weak Freeze-Out" ($\lambda \lesssim H$) sets $\frac{n}{p} \approx \frac{1}{6} \Rightarrow$ some n decay $\Rightarrow \frac{n}{p} \approx \frac{1}{7}$
and all neutrons go into alpha particles:

$$
\frac{n}{p} \approx \left.\frac{\lambda_{p\to n}}{\lambda_{n\to p}}\right|_{\mathrm{WFO}} \Rightarrow Y_p({}^4He)
$$

$$
Y_p \approx \frac{2\mathrm{n}}{\mathrm{n}+\mathrm{p}} = \frac{2\mathrm{n}/\mathrm{p}}{\mathrm{n}/\mathrm{p}+1} \approx 0.25!!
$$

Constraining Experiments with the Early Universe

A light sterile neutrino ($m \ll 1$ MeV) that mixes with actives can have dire consequences for the standard picture of cosmology and the early universe.

A sterile neutrino that mixes with active flavors can be brought into thermal contact with the primordial plasma either through direct oscillation conversion $\nu_\alpha \to \nu_s$, or through a resonant conversion. This can affect the synthesis of the light elements in the early universe in two ways:

- Increasing the total energy density of the early universe — This happens since some of the thermal energy in the primordial plasma can be stored in the degree of freedom made available by the ν_s. The increase in energy density increases the expansion rate, the neutron-to-proton ratio, and thus *increases* the universal abundance of ^{4}He.

- Altering the weak n $\rightleftharpoons$ p rates — This happens when $\nu_e \rightleftharpoons \nu_s$ mix resonantly, causing an asymmetry in ν_e vs. $\bar{\nu}_e$. Such an asymmetry directly affects the weak rates that set the neutron-to-proton ratio at nucleosynthesis, and can *either increase* **or** *decrease* the universal ^{4}He abundance.

Both of these effects have been shown to cause undesirable consequences to the primordial helium abundance, beyond the observed bounds (Shi, Schramm & Fields, 1993; Enqvist, Kainulainen & Thomson, 1992) These bounds are shown in the first overlay in the figure to the right.

Also, resonant conversion can be enhanced if the generation of lepton number is chaotic. Horizons previously out of contact with each other can develop different signs of L_{ν_α}. These regions then come into causal contact, and neutrinos that travel from one region to another that have opposite signs of L_{ν_α} can undergo further adiabatic resonance conversion into steriles. These bounds are shown in the second overlay to the right.

Specific 4 Neutrino Models and BBN

BBN Constraints from direct-oscillation (non-resonant) conversion:

$$\delta m^2 \sin^4 2\theta \lesssim 10^{-7}\ \mathrm{eV}^2$$
$$\delta m^2_{es} \sin^4 2\theta_{es} \lesssim 10^{-9}\ \mathrm{eV}^2$$

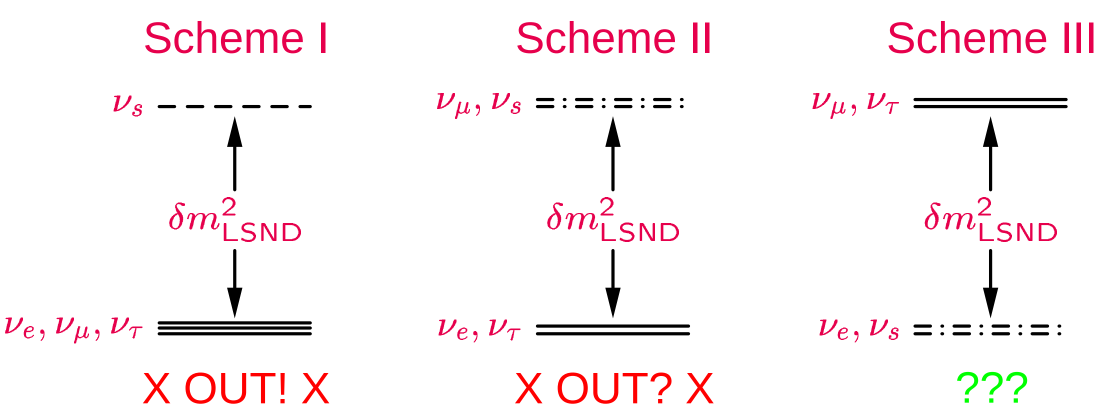
(Enqvist *et al.* 1992; Shi, Schramm & Fields 1993)

Lepton-domain driven BBN:

$-\delta m^2 \sin^2 2\theta \lesssim 10^{-4}\ \mathrm{eV}^2$ for vacuum solar sol'n *and* $\delta m^2 < 4\ \mathrm{eV}^2$

$\sin^2\theta \lesssim 10^{-11}$ for MSW solar sol'n *or* $\delta m^2 > 4\ \mathrm{eV}^2$

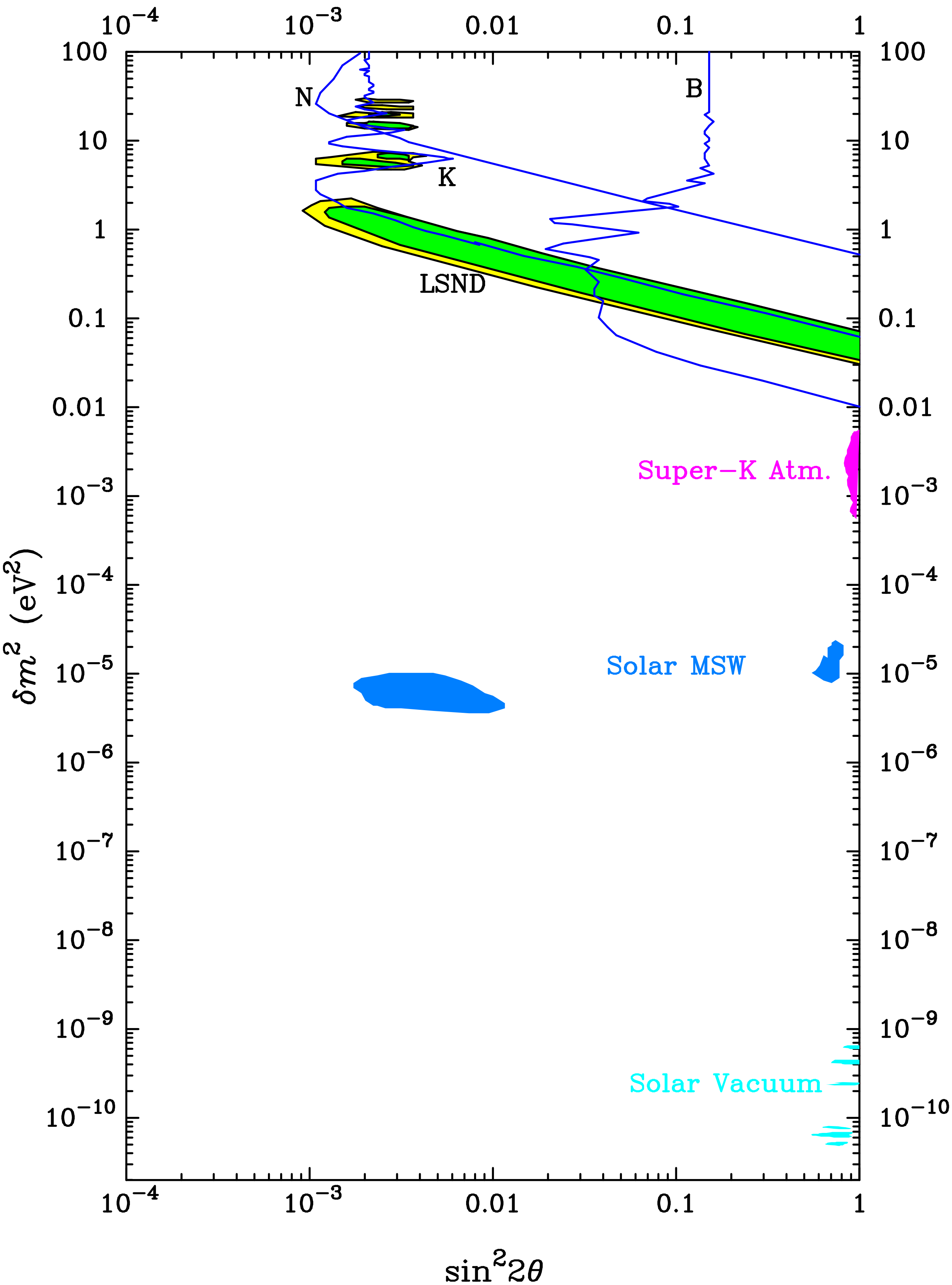
(Shi & Fuller 1999; Abazajian, Fuller and Shi 2000)



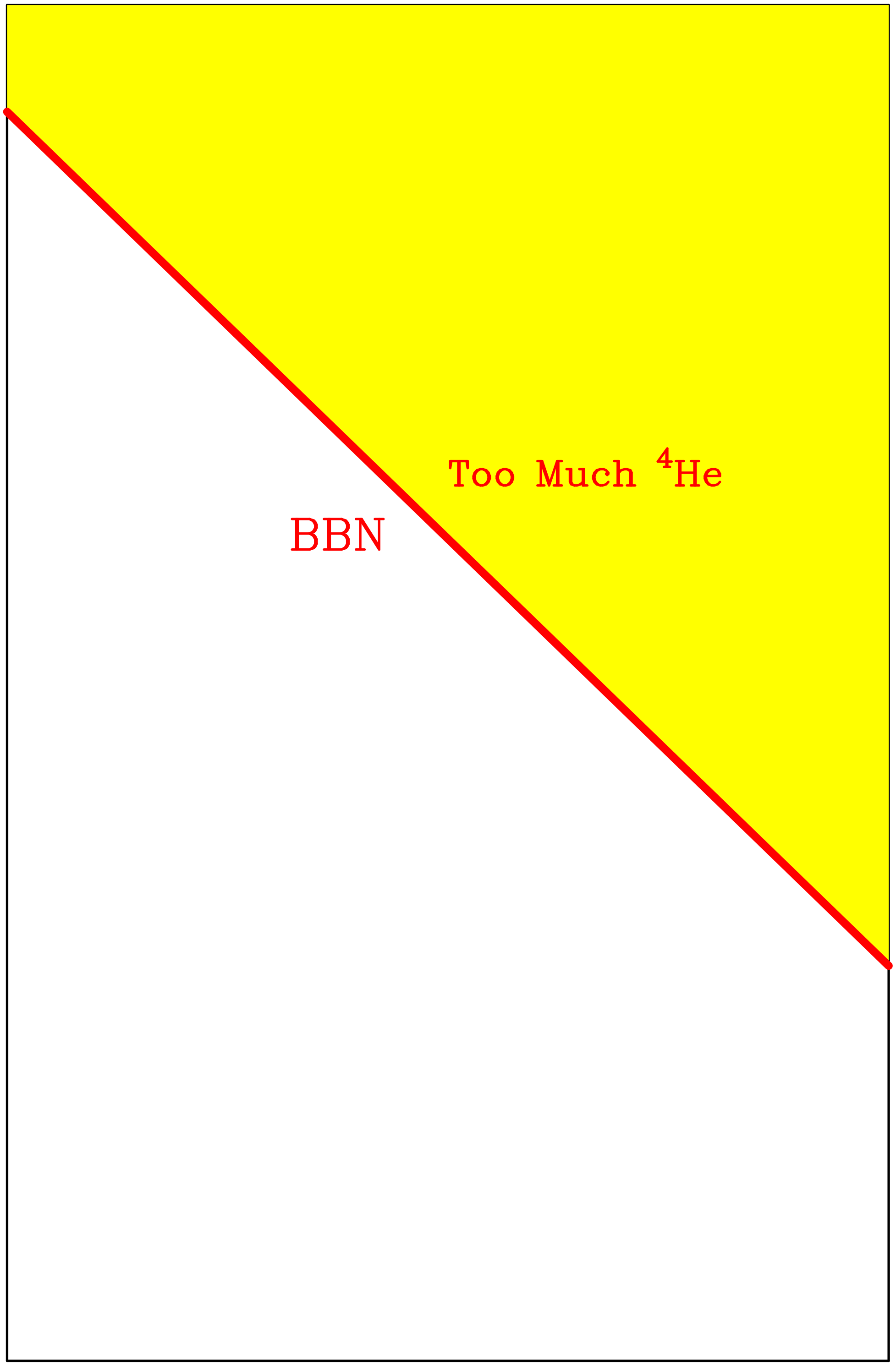
Scheme I: $\delta m^2_{e\mu,s}$, $\sin^2 2\theta_{e\mu,s}$ too large! (indirect LSND mixing). Also disfavored by a combination of constraints from LSND, Super-K, and CDFS (Bilenky *et al.* 1999). However, a new analysis by LSND may make this model viable (Barger *et al.* 2000), but still inconsistent with BBN.

Scheme II: $\delta m^2_{\mu s}$, $\sin^2 2\theta_{\mu s}$ too large (Super-K Atmospheric Solution).

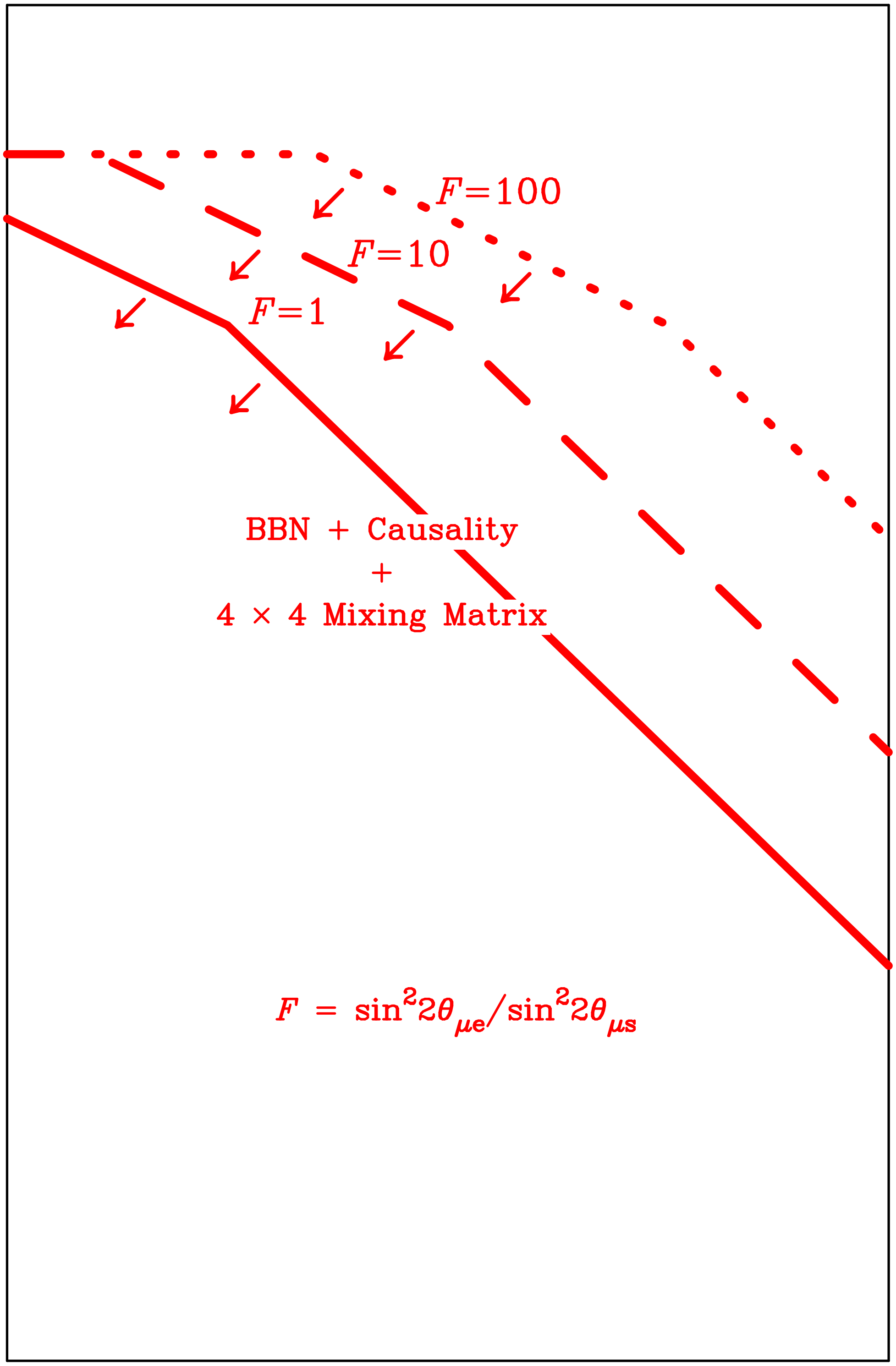
Scheme III: Okay if $F \equiv \frac{\sin^2 2\theta_{e\mu}}{\sin^2 2\theta_{es}} \gtrsim 100$ for vacuum solar solution, or $F \gtrsim 10^7$ for MSW solution.



N
B
K
LSND
Super−K Atm.
Solar MSW
Solar Vacuum
δm^2 (eV^2)
$\sin^2 2\theta$



(Enqvist *et al.*, 1992)
(Shi, Schramm and Fields, 1993)



(Abazajian, Fuller and Shi, 2000)